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Research paper

Analyzing VaR: The Case for Wavelet Methods in the Moroccan Food Industry

LAABIDI Khalid[1,*], EL AALLAOUI Mohamed [1]

[1] *Laboratory of modelling applied to economics and management*
Faculty of Law, Economics and Social Sciences of Ain Sebaâ
University Hassan II, Casablanca, Morocco

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ABSTRACT

The quantification of credit portfolio losses using the wavelet approach offers an innovative methodology for assessing the financial risks associated with credit. This approach uses advanced mathematical techniques to analyse temporal fluctuations in credit data. In terms of quantifying losses, the wavelet approach allows the decomposition of loss time series into different time scales. This makes it possible to identify short- and long-term trends as well as irregular variations. By analysing these scales, analysts can better understand the dynamics of credit losses and identify the underlying factors that contribute to fluctuations. To quantify credit portfolio losses, the cumulative loss function is approximated by a finite combination of wavelet basis functions by computing the coefficients of the wavelet approximation (WA). Wavelet approximation is an accurate, robust and fast method that enables VaR to be estimated much more quickly than with other loss quantification methods, such as the Monte Carlo MC method.

1 Introduction

This template provides authors with most of the formatting specifications needed for preparing electronic versions of their papers. All standard paper components have been specified for three reasons: (1) ease of use when formatting individual papers, (2) automatic compliance to electronic requirements that facilitate the concurrent or later production of electronic products, and (3) conformity of style throughout a conference proceedings. Margins, line spacing, and type styles are built-in; examples of the type styles are provided throughout this document and are identified in italic type, within parentheses, following the example. Some components, such as multi-leveled equations, graphics, and tables are not prescribed, although the various table text styles are provided. The formatter will need to create these components, incorporating the applicable criteria that follow.

In the context of credit risk management, particular attention is paid to the distribution of losses in the portfolio. Since portfolio loss is often modelled as the sum of random variables, the predominant task is to evaluate the probability density function (PDF) associated with this loss distribution. This PDF of a sum of random variables is equivalent to the convolution of the respective PDFs of the loss distributions of the individual assets. Performing this evaluation analytically poses a considerable challenge and requires significant computational intensity. In general, this approach is not feasible for realistically sized portfolios [1].

*Corresponding author. Email: Khalidlaabidi6@gmail.com

Monte Carlo simulation is a common approach to assessing the risk of a credit portfolio. However, this method becomes time-consuming as the size of the portfolio increases. In many circumstances, calculations can become difficult to perform, especially given the frequent need for financial institutions to readjust their credit portfolios.

Wavelets offer the flexibility to capture complex and changing patterns in time series. They can accommodate non-linear variations and discontinuous behaviour, which can be particularly useful in the context of credit losses, they can help identify the points at which losses in the credit portfolio undergo significant changes. This can help identify periods of increased volatility or significant credit events.

Wavelet approximation is increasingly emerging in financial applications. Much of the research that has been done by researchers such as: "High-frequency wavelet analysis of financial time series" (2007) - This research by Huang, N. E., et al. examined the application of wavelets to high-frequency financial time series. "A wavelet analysis of MENA stock market risk: An application of the Continuous Wavelet Transform" (2018) - This research by Charfeddine, L., et al. applied the Continuous Wavelet Transform to the analysis of Middle Eastern and North African stock market risk.

2 Methodology

2.1 Credit portfolio losses

A portfolio's estimated losses will be calculated as expected losses (EL) by modelling exposure at default (EAD), probability of default (PD) and Loss given default (LGD). For the banker, credit risk or counterparty risk is defined as "the risk that the customer will not honor his financial commitment. of a customer's failure to meet a financial commitment, in most cases a loan repayment, loan repayment". In a broader sense, counterparty risk also refers to the risk of deterioration in the borrower's financial health, which reduces the probability of repayment: default risk. repayment: default risk. After the expected losses until certain confidence level is called economic capital. The expected losses can be calculated with the following formula:

$$\mathcal{L} = \mathrm{EAD} \times \mathrm{PD} \times \mathrm{LGD}$$

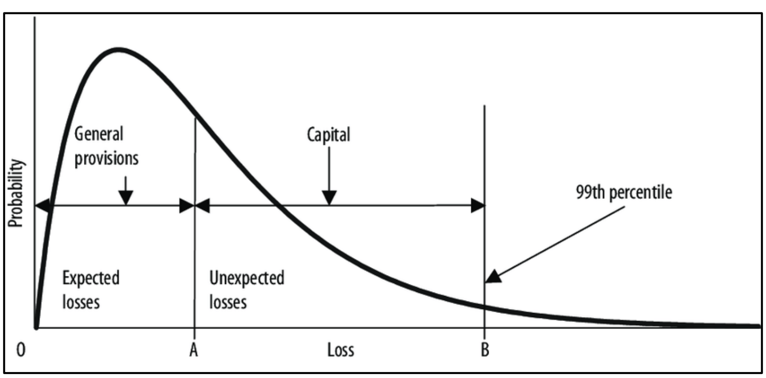


Figure 2 : Distribution of credit losses[2]

Exposure at default (EAD) is the total value a bank is exposed to when a loan defaults. Using the internal ratings-based (IRB) approach, financial institutions calculate their risk. Banks often use internal risk management default models to estimate respective EAD systems. Probability of default (PD) is a financial term describing the likelihood of a default over a particular time horizon. Loss given default or (LGD) is the share of an asset that is lost if a borrower defaults. It is a common parameter in risk models and also a parameter used in the calculation of economic capital, expected loss or regulatory capital under Basel II for a banking institution.

For our research, we calculated these three parameters for three food industry companies in Morocco, based on the activity reports and bulletins generated by the Casablanca stock exchange.

2.2 Value at Risk

Let us consider a portfolio with N obligors and let F be the cumulative distribution function of losses $\mathcal{L}$. Without loss of generality

$$F(x) = \begin{cases} \bar{F}(x), & \text{if } 0 \leqslant x \leqslant 1, \\ 1, & \text{if } x > 1, \end{cases} \tag{1}$$

Value at Risk (VaR)[3] is a measure commonly used in finance to assess investment risk. It indicates the maximum amount of loss expected (in monetary terms or as a percentage) for an investment or portfolio, with a certain level of confidence and over a certain time horizon.

Let X be loss distribution, at a level $\alpha \in (0,1)$ the VaR is the smallest number y such as that the probability that $Y := -X$ below $1-\alpha$. Mathematically,

$$\begin{aligned} \mathrm{VaR}_\alpha(X) &= inf\{l \in \mathbb{R}: \mathbb{P}(\mathcal{L} \leq l) \geq \alpha\} = inf\{l \in \mathbb{R}: F_{\mathcal{L}}(l) \geq \alpha\} \\ &= -inf\{x \in \mathbb{R}: F_X(x) > \alpha\} = F_Y^{-1}(1-\alpha) \end{aligned} \tag{2}$$

This is the measure selected in the Basel II Accord for calculating capital requirements. This means that a bank that manages its risks in accordance with Basel II must set aside an amount of capital to cover any extreme losses.[4]

2.3 Wavelet approximation

There are two functions that play a major role in wavelet analysis: the scaling function φ and the wavelet ψ. These two functions create a family of functions that can be used to decompose or reconstruct a signal. To emphasize the "marriage" involved in building this "family", ϕ is sometimes called the "parent wavelet" and ψ the "mother wavelet". The Haar scaling function is defined as:

$$\phi_H(x) = \begin{cases} 1, & \text{if } 0 \leq x < 1 \\ 0, & elsewhere. \end{cases} \tag{3}$$

The Haar mother wavelet function can be described as:

$$\psi_H(x) = \begin{cases} 1, & \text{for } 0 \leq x < \frac{1}{2} \\ -1, & \text{for } \frac{1}{2} \leq x < 1 \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

The drawback with the Haar decomposition is that both father and mother wavelet are discontinuous, and as a result, it provides only crude approximations to some continuous functions.

By the Haar wavelet approximation, the approximation of the cumulative distribution function is:

$$\bar{F}_m(x) = \sum_{k=0}^{2^m-1} c_{m,k}\phi_{m,k}(x) \tag{5}$$

With $c_{m,k}$ the approximation coefficient and *m* the level of resolution:

$$c_{m,k} = \langle f, \phi_{m,k} \rangle = \int_{-\infty}^{+\infty} f(x)\phi_{m,k}(x)\mathrm{d}x \tag{6}$$

And $\phi_{j,k}$ translated version:

$$\phi_{j,k} = 2^{\frac{j}{2}}\phi(2^j x - k) \tag{7}$$

In the context of one-factor model [5], it is important to remember that if the systematic factor Y remains constant, failures occur independently. In effect, the only remaining source of uncertainty is idiosyncratic risk.

The moment generating function (MGF) is conditional on Y is thus given by the product of each obligor's MGF:

$$M_L(s;Y) \equiv \mathbb{E}(\mathrm{e}^{-sL} \mid Y) = \prod_{n=1}^{N} \mathbb{E}(\mathrm{e}^{-sE_nD_n} \mid Y)$$
$$= \prod_{n=1}^{N} [1 - p_n(y) + p_n(y)\mathrm{e}^{-sE_n}]. \quad (8)$$

With E_n = *EAD* of *n* obligors, D_n the indicator of default and P_n = *PD* of *n* obligors.

If *f* is the probability density function of the loss function, then the unconditional moment generative function is:

$$M_L(s) \equiv \mathbb{E}(\mathrm{e}^{-sL}) = \int_0^{+\infty} \mathrm{e}^{-sx} f(x)\mathrm{d}x = \tilde{f}(s) \quad (9)$$

2.4 Calculation algorithm

We can prove that

$$0 \le c_{m,k} \le 2^{-m/2}, k = 0,1,\dots,2^m - 1$$

And

$$0 \le c_{m,0} \le c_{m,1} \le \cdots \le c_{m,2^m-1}.$$

Based on an approximation at a certain level of resolution m, VaR can now be calculated quickly using *m* coefficients, due to the compact support of the basic functions:

$$\bar{F}(\mathrm{VaR}_\alpha) \simeq \bar{F}_m(\mathrm{VaR}_\alpha) = 2^{\frac{m}{2}} \cdot c_{m,\bar{k}}, \quad (10)$$

For $\bar{k} \in \{0,1,\dots,2^m - 1\}$, we search VaR_α by going through the following steps: first, we compute $\bar{F}_m\left(\frac{2^{m-1}}{2^m}\right)$, if $\bar{F}_m\left(\frac{2^{m-1}}{2^m}\right) > \alpha$ then we compute $\bar{F}_m\left(\frac{2^{m-1}-2^{m-2}}{2^m}\right)$, after *m* steps storing the $\bar{k}$ value such that $\bar{F}_m\left(\frac{\bar{k}}{2^m}\right)$ is the closest value to α in our m resolution approximation.[6,7]

In fact, due to the stepped shape of the Haar wavelets approximation, $\bar{F}_m(\xi) = \bar{F}_m\left(\frac{\bar{k}}{2^m}\right)$, for all $\xi \in \left[\frac{\bar{k}}{2^m}, \frac{\bar{k}+1}{2^m}\right)$. In what follows let us take, $\mathrm{VaR}_\alpha^{W(m)} = \frac{2\bar{k}+1}{2^{m+1}}$, the middle point of this interval, as the VaR value computed by means of this wavelet algorithm at scale m.

3 Data and results

Our objective is to calculate VaR using two approaches, the standard Monte Carlo approach and the advanced Haar wavelet approach. We will use data from Moroccan companies in the field of food industry for a recent period *Lesieur, Cosumar and Labelvie*. We will generate a Matlab code that calculates the VaR by the two approaches, also calculating the time needed to calculate the VaR by each approach, and also the error between each approach for a confidential level 95%. Our calculations and programming are done on a computer i5 4th generation 512 GB SSD, 16 GB RAM.

	N	PD	EAD	LGD
Lesieur	100	0,15	0,3$rand$	0,3+0,3$rand$
Cosumar	200	0,15	0,52$rand$	0,63$rand$
Labelvie	100	0,09	0,5$rand$	0,3$rand$

Figure 3: research data combined by us

Our data is collated from quarterly financial reports, studying the balance sheet to determine the amounts of long-term credit and accounting ratios based on the net income and financial of each company, setting the maturity T to one year and by applying the model of Merton, we calculated the PD, and also based on the rating agencies Moody's for the quantification of parameter LGD, and also, we used the monthly editions and bulletins of sessions issued by the Casablanca Stock Exchange, we calculated EAD. And using MATLAB, we generated a program that calculates VaR in parallel using the Monte Carlo method and Wavelet approximation. For the number N, we have chosen a number which is not larger, because the data is not available for all durations, and also the characteristics of our machine are not sufficient to make a calculation for a large enough number.

$\alpha = 5\%$	$\mathrm{VaR}_{\alpha}^{MC}$	$\mathrm{VaR}_{\alpha}^{WA}$	$\mathrm{Time}^{\ MC}$	Time^{WA}	$\mathrm{RE}(\alpha, m)$
$Lesieur$	0,9023	0,8994	11,9879	1,4329	0,0032
$Cosumar$	0,7431	0,7412	18,6032	2,7149	0,0026
$Labelvie$	0,1191	0,1221	18,0531	2,7249	0,0251

Figure 4: Programme results

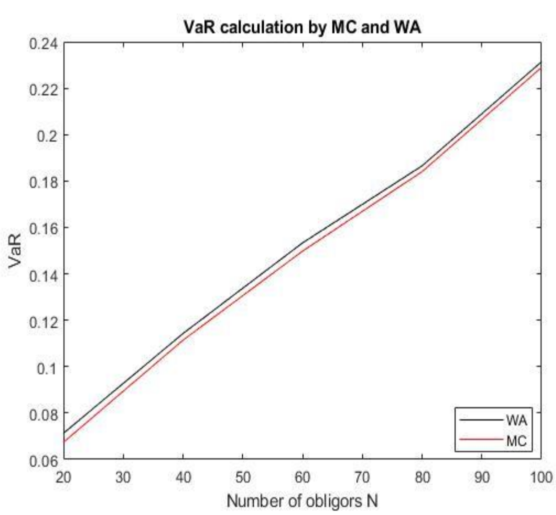


Figure 5 : VaR by MC and WA (Lesieur)

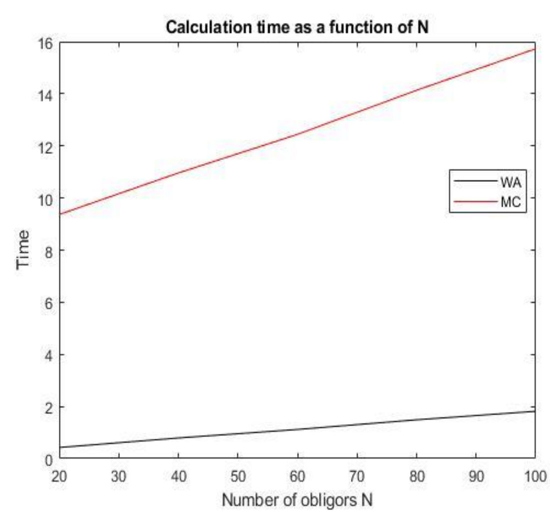
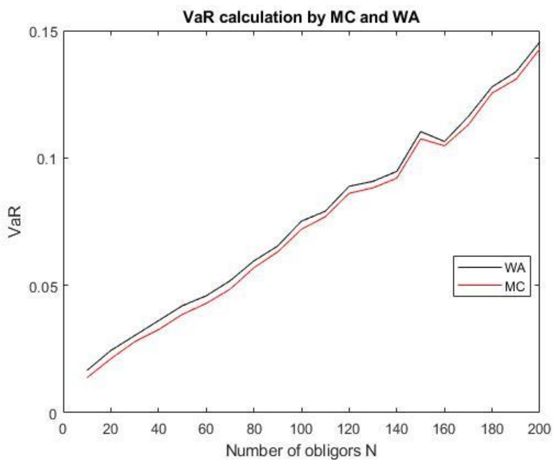


Figure 6 : Calculation time (Lesieur)



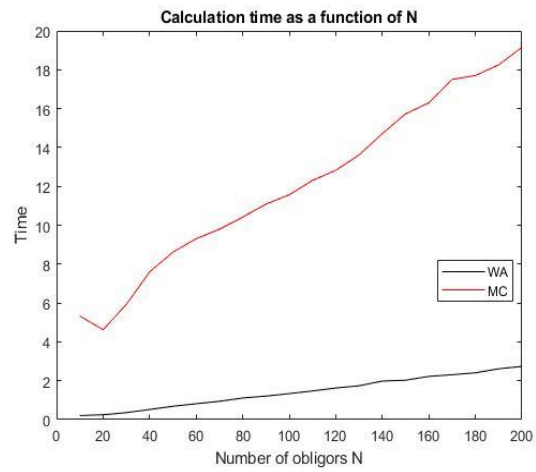


Figure 7: VaR by MC and WA (Cosumar)

Figure 8: Calculation time (Cosumar)

4 Discussions

For a risk level of 5%, we calculated the VaR of portfolios with different parameters using Matlab. Using the standard approach (MC) we found reasonable values, and using the haar wavelet approximation we also found a value approximating that of MC, which does not reflect a major difference between the two approaches.

The most important point is the calculation time, graphs 4 and 6 show the difference between MC and WA, that is the time to calculate the VaR of a portfolio of 100 and 200 bonds, for example for N=200, the VaR requires 19 seconds for the calculation with Monte Carlo, against the wavelets just 3 seconds.

5 References

1. Takano, Y. and Hashiba, J., A novel methodology for credit portfolio analysis: numerical approximation approach, (2008). Available online at: www. defaultrisk.com
2. Powell, Andrew & Majnoni, Giovanni "Reforming Bank Capital Requirements: Implications of Basel II for Latin American Countries'' 10.1353/eco.2005.0019
3. Martin, R. and Ordova´s, R., An indirect view from the saddle. Risk, (2006), October, 94–99.
4. Foellmer, Hans; Schied, Alexander (2004). Stochastic Finance. de Gruyter Series in Mathematics. Vol. 27. Berlin : Walter de Gruyter. pp. 177–182. ISBN 978-311-0183467. MR 2169807.
5.
6. Joseph J. Masdemont & Luis Ortiz-Gracia (2014) Haar wavelets-based approach for quantifying credit portfolio losses, Quantitative Finance, 14:9, 1587-1595, DOI: 10.1080/14697688.2011.595731
7. Abate, J., Choudhury, G.L. and Whitt, W., An introduction to numerical transform inversion and its application to probability models. In Computational Probability, edited by W.K. Grassman, pp. 257–323, (2000) (Kluwer: Norwell, MA).
8. Ahn, J., Kang, S. and Kwon, Y., A flexible inverse Laplace transform algorithm and its application. Computing, (2003), 71, 115–131.